2 points!

1 point!

Difference Quotient, **Avg Rate of Change**	$Q = \frac{f(a+h) - f(a)}{h}$ $A(x) = \frac{f(b) - f(a)}{b - a}$	**SAME.** Used to calculate the **slope of the secant** line between two points on a graph of function f.	The **slope of a secant line going through 2 points on the curve**, averaging a range of values. This is contrasting with the *Instantaneous Rate of Change which is the slope of the tangent line touching at <u>one point</u>, evaluating at a single, isolated moment on the curve.*
Instantaneous Rate of Change	Same as **derivative**! Same as **tangent line slope**! $f'(x)$ at a point, x	It is the change in rate at a particular instant, and is the **derivative**, or **tangent line slope**	speed = $s(t)$ velocity = $v(t) = s'(t)$ acceleration = $a(t) = v'(t) = s''(t)$ **Speed** = function **Velocity** = first derivative of speed **Acceleration** = first derivative of velocity and second derivative of speed Let $y = x^2 - 2$. Find the instantaneous rate of change of y with respect to x at the point x = 4. $y = f(x) = x^2 - 2$ $f'(x) = 2x$ $f'(4) = 2(4) = \boxed{8}$
Constant Rule	$f(x) = c$ $\mathbf{f'(x) = 0}$	The derivative of a constant = **ZERO**	**The derivative of any constant that is being added or subtracted = 0.** If the constant is being MULTIPLIED or DIVIDED, this rule does NOT APPLY. See Constant Rule Multiple. $y = x^2 - 2$ $y' = \frac{d}{dx} x^2 - \frac{d}{dx} 2$ $y' = 2x - 0$ $\boxed{y' = 2x}$
Power Rule	$f(x) = x^n$ $\mathbf{f'(x) = n \bullet x^{n-1}}$	Multiply the variable's exponent **n** by the function **(moves to front)** and subtract 1 from the exponent **n**.	$y = x^3$ $y' = 3 \cdot x^{3-1}$ $\boxed{y' = 3x^2}$ $y = 4x^5$ $y' = 5 \cdot 4x^{5-1}$ $\boxed{y' = 20x^4}$ $y = 2x^3 - 6x^2 + 3$ $y' = 3 \cdot 2x^{3-1} - 2 \cdot 6x^{2-1} + 0$ $\boxed{y' = 6x^2 - 12x}$
Sum Rule	$\frac{d}{dx} f(x) + g(x)$ $\mathbf{\frac{d}{dx} f(x) + \frac{d}{dx} g(x)}$	The derivative of a sum of functions is equal to the sum of their individual derivatives.	$y = 2x^3 + 6x^2 + 3$ $y' = \frac{d}{dx} 2x^3 + \frac{d}{dx} 6x^2 + \frac{d}{dx} 3$ $y' = 3 \cdot 2x^{3-1} + 2 \cdot 6x^{2-1} + 0$ $\boxed{y' = 6x^2 + 12x}$
Difference Rule	$\frac{d}{dx} f(x) - g(x)$ $\mathbf{\frac{d}{dx} f(x) - \frac{d}{dx} g(x)}$	The derivative of a difference of functions is equal to the difference of their individual derivatives.	$y = 2x^3 - 6x^2 - 3$ $y' = \frac{d}{dx} 2x^3 - \frac{d}{dx} 6x^2 - \frac{d}{dx} 3$ $y' = 3 \cdot 2x^{3-1} - 2 \cdot 6x^{2-1} - 0$ $\boxed{y' = 6x^2 - 12x}$
Constant Rule Multiple	$f(x) = a \bullet g(x)$ $\mathbf{f'(x) = a \bullet \frac{d}{dx} g(x)}$	The derivative of a constant multipled by a function is equal to the constant multiplied by the derivative of the function. The constant goes to the front and hangs out.	$y = 8x^5$ $y' = 8 \cdot \frac{d}{dx} x^5$ $y' = 8 \cdot 5x^4$ $\boxed{y' = 40x^4}$ $y = 5x^3 + 2x^2 - 6x$ $y' = 5 \cdot \frac{d}{dx} x^3 + 2 \cdot \frac{d}{dx} x^2 - 6 \cdot \frac{d}{dx} x$ $y' = 5 \cdot 3x^2 + 2 \cdot 2x - 6 \cdot 1$ $\boxed{y' = 15x^2 + 4x - 6}$
Product Rule	$a(x) = f(x) \bullet g(x)$ $\mathbf{a'(x) = f'g + g'f}$	The derivative of a product of functions is equal to: $f'(x) \bullet g(x) + g'(x) \bullet f(x)$	$y = (x^2 + 2)(3x^3 - 5x)$ $f(x) = x^2 + 2$ $g(x) = 3x^3 - 5x$ $f'(x) = 2x + 0$ $g'(x) = 9x^2 - 5$ $f' \cdot g + g' \cdot f$ $= 2x \bullet (3x^3 - 5x) + (9x^2 - 5)(x^2 + 2)$ $= \boxed{15x^4 + 3x^2 - 10}$
Quotient Rule	$a(x) = \frac{f(x)}{g(x)}$ $\mathbf{a'(x) = \frac{f'g - g'f}{g^2}}$	The derivative of a quotient of functions is equal to: $f'(x) \bullet g(x) - g'(x) \bullet f(x)$ all divided by $g(x)^2$	$y = \frac{5x^2}{4x + 3}$ $f(x) = 5x^2$ $g(x) = 4x + 3$ $f'(x) = 10x$ $g'(x) = 4$ $\frac{f' \cdot g - g' \cdot f}{g^2}$ $y' = \frac{10x \bullet (4x + 3) - 4 \bullet (5x^2)}{(4x + 3)^2}$ $\boxed{y' = \frac{20x^2 + 30x}{(4x + 3)^2}}$
Chain Rule	$a(x) = f(\ g(x)\ )$ $\mathbf{a'(x) = f'(g(x)) \bullet g'(x)}$	When taking the derivative of a composite function, first take the derivative of the outer function ***f(x)*** then multiply times the derivative of the inner function, ***g(x)***	$y = \sin^3 x \rightarrow y = (\sin x)^3$ $y' = 3(\sin x)^{3-1} \bullet \frac{d}{dx}(\sin x)$ $\boxed{y' = 3 \sin^2 x \cos x}$ $y = \cos(5x^2)$ $y' = -\sin(5x^2) \cdot \frac{d}{dx} 5x^2$ $y' = -\sin(5x^2) \cdot 10x$ $\boxed{y' = -10x \sin(5x^2)}$

#1 tip: If something does not make sense, google "purpose of ___" and/or "what is ___". Watch videos, ask questions, DM ME!

Inverse Function Theorem

$$g'(x) = \frac{1}{f'(g(x))}$$

1. Find the inverse of g(x) by switching x and y and solving for y. The inverse $g^{-1}(x)$ is our f(x).
2. Find the derivative of the inverse f(x) = f′(x).
3. Plug in the original g(x) function into the new f′(x) as in the formula, 1/f′(g(x)) and simplify.

Limits

*Remember: **slope of tangent line** and **derivative** are the SAME THING*

$$m_{\text{tangent line}} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

$$f'(x)$$

Calculus is the study of limits. When you see limit, think "approaching." So if there is a hole or discontinuity in the graph, you can "approach" from either side of the discontinuity and see what value the function is getting closer and closer to.

Dx Trig Functions

$\frac{d}{dx}$ for all

$\sin(x) = \cos(x)$

$\cos(x) = -\sin(x)$

$\tan(x) = \sec^2(x)$

$\cot(x) = -\csc^2(x)$

$\sec(x) = \sec(x)\tan(x)$

$\csc(x) = -\csc(x)\cot(x)$

Definition of Derivative

$$f'(x) = \lim \frac{f(x+h) - f(x)}{h}$$

L'Hopital's Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Derivatives DO NOT EXIST on corners, discontinuities (hole, jump) or vertical tangents, basically if you cannot draw a tangent line.

Derivatives of Inverse Functions

$\sin^{-1} x = \frac{1}{\sqrt{1-(x)^2}}$ $\quad$ $\sec^{-1} x = \frac{1}{|x|\sqrt{(x)^2-1}}$

$\cos^{-1} x = \frac{-1}{\sqrt{1-(x)^2}}$ $\quad$ $\csc^{-1} x = \frac{-1}{|x|\sqrt{(x)^2-1}}$

$\tan^{-1} x = \frac{1}{1+(x)^2}$ $\quad$ $\cot^{-1} x = \frac{-1}{1+(x)^2}$

Slope equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point Slope Form

$y - y_1 = m(x - x_1)$

Slope Intercept Form

$y = mx + b$

Factoring

$a^2 - b^2 = (a+b)(a-b)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Derivatives of Log Functions

$\frac{d}{dx}$ for all

$a^x = a^x \ln(a)$

$e^x = e^x$

$\ln(x) = \frac{1}{x}, x > 0$

$\ln|x| = \frac{1}{x}, x \neq 0$

$\log_a(x) = \frac{1}{x \ln a}, x > 0$

Implicit Differentiation solve for y' or $\frac{dy}{dx}$

This allows you to find the derivative of y with respect to x without having to solve the given equation for y.

Each time you differentiate y, multiply the derivative by **dy/dx** or **y′** (these are the same thing, "the derivative of y"), then solve the equation for y′. DONE.

$$6x^2 + 3y^2 = 12$$

$$2 \cdot 6x^{2-1} + 2 \cdot 3y^{2-1} \cdot \frac{dy}{dx} = 0$$

$$12x + 6y\frac{dy}{dx} = 0$$

$$-12x \qquad -12x$$

$$\frac{6y\frac{dy}{dx}}{6y} = \frac{-12x}{6y}$$

$$\boxed{\frac{dy}{dx} = -\frac{2x}{y}}$$

Linear Approximation $L(x) = f(a) + f'(a)(x - a)$

Used to approximate the value of a function at a particular point, a.

1) Find f(a)
2) Find f′(a)
3) Plug in to L(x)

$$f(x) = \sin^2 x, \quad a = 0$$

$$L(0) = f(0) + f'(0)(x-0)$$

$$f(0) = \sin^2 0 = \boxed{0}$$

(1,0)

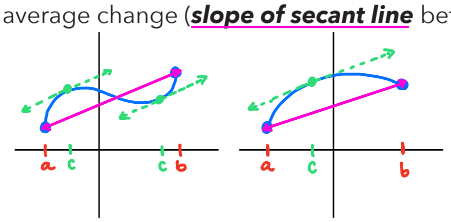
$$f'(x) = 2\sin x \cos x$$

$$f'(0) = 2\sin(0)\cos(0) = \boxed{0}$$

$$\boxed{L(0) = 0 + 0(x-0) = 0}$$

Mean Value Theorem

The slope of the tangent line (*same as f′(c)*) at c is the same as the secant slope between f(a) and f(b). At some point, the instantaneous change (***tangent line*** at c) will be the same as the average change (***slope of secant line*** between a and b).



The slope of the tangent line at c and the slope of the secant line from a to b are the same, or parallel.

To find values of c where the **tangent** and **secant** are the same, find **f′(x)** and **average change** and set them equal to each other, i.e. **f′(x) = [f(b) - f(a)]/(b - a)**

Increasing/Decreasing Extrema Max/Min

Extrema occur when there is a change in direction (mountain or valley). The CRITICAL POINT is located at the "vertex" of the change of direction, where the slope of the tangent line = 0. To find critical points, find the derivative of the function, then set it = 0 and solve. Then plug in this x value to the original function to find the location of the change of direction. Bigger = max, Smaller = min

$$y = x^2 + \frac{2}{x} \quad \text{over } [1, 4]$$

$$y' = 2x + 2 \cdot -1x^{-2} = 2x - \frac{2}{x^2}$$

$$2x - \frac{2}{x^2} = 0$$

$$x^2 \cdot 2x = \frac{2}{x^2} \cdot x^2$$

$$x^3 = 1$$

$$\boxed{x = 1}$$

$$\boxed{f(1) = 3}$$

always check ENDPOINTS!

$[1, 4]$

$f(4) = 16 + \frac{1}{2}$

$\boxed{f(4) = 16.5}$

Absolute maximum **x=4, y=16.5**
Absolute minimum **x=1, y=3**

Derivatives give you rate of change of function. It is the ratio of "differentials" ie (dy/dx) ||| A **differential** is the change in a *single variable*. Ex: Find the differential, means **find y′ (or dy)**