Formula Sheet: First midterm 2203, Fall 2012

1) Relativity: space and time

Transforms from S to S': : **$\gamma = 1/(1-v_o^2/c^2)^{1/2}$** v_o in along the x axis.

$x' = \gamma(x - v_o t)$

$y' = y$

$z' = z$

$z' = z$

$t' = \gamma(t - (v_o/c^2)x)$

$v'_x = (v_x - v_o)/(1 - v_x v_o/c^2)$

$v'_y = v_y/\gamma(1 - v_x v_o/c^2)$

$v'_z = v_z/\gamma(1 - v_x v_o/c^2)$

Change sign to go from S' to S:

Time dilation $\Delta t = \gamma \Delta t_0$: Δt_0 Proper measurement of time.

Length contraction $\Lambda = \Delta L_0/\gamma$: L_0 proper measurement of length.

2) Relativistic Mechanics

$E' = \gamma[E - (v_o/c)(pc)]$

$\mathbf{p} = \gamma m\mathbf{v}$ $\mathbf{p}c = \gamma mc^2(\mathbf{v}/c)$

$\mathbf{F} = d\mathbf{p}/dt$

$E^2 = p^2c^2 + (mc^2)^2$: $\sqrt{E^2 - (mc^2)^2}$

$\mathbf{v} = \mathbf{p}c^2/E$

$p' = \gamma[p - (v_o/c^2)/E]$

E_K(kinetic energy) $= mc^2(\gamma - 1)$

E (total energy) $= \gamma mc^2$

E=pc for massless particle

$E = E_K + mc^2$

Relativistic Doppler shift $f'/f = \gamma[1 - (v/c)\cos\theta]$ or $f'/f = [(1-v/c)/(1+v/c)]^{1/2}$ (for $\theta = 0^o$)

3) Atoms

Notation $^A_Z Z_N$ Where A=Z+N with Z being the number of protons and N the number of neutrons. An example is Carbon with 6 neutrons and 6 protons $^{12}_6 C_6$

Kinetic theory pV=nRT or pV=Nk_BT, with k_B the Boltzmann's constant and R the universal gas constant.

$k_B = R/N_A$ N_A is Avogadro's number

average kinetic energy $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_B T$

4) Quantization of light:

Photon has energy hf

E = hf, $E = \hbar\omega$, with $\omega = 2\pi f$

E=pc

$E = hc/\lambda = 1240/\lambda$ eV nm

Photoelectic effect

$K_{max} = hf - e\phi$

$e\phi$ is work function

Compton Scattering

$\lambda' - \lambda = \lambda_c(1 - \cos\theta)$

$\lambda_c = h/m_e c = 0.00246$ nm

Group velocity $v_g = \frac{d\omega}{dk}$, Phase velocity $v_p = \frac{c^2}{v}$ and $v = v_g$

Bragg Scattering $2d\sin\theta = n\lambda$

5) Quantization of Atomic Energy levels: Bohr Atom

Rydberg series series $\frac{1}{\lambda} = R[\frac{1}{n^2_f} - \frac{1}{n^2_i}]$, i is the initial state and f the final state: R is Rydberg constant.

Bohr Model: The angular momentum is quantized $L = mvr = n\hbar$ or $2\pi r_n = n\lambda$ yielding the following equations:

$r_n = \frac{n^2\hbar^2}{ke^2 m}$ where the Bohr orbit radius is defined $a_0 = \frac{\hbar^2}{ke^2 m} = 0.05292nm$

$E_n = -\frac{m}{2n^2}(\frac{ke^2}{\hbar})^2$ giving $E_n = -\frac{13.6eV}{n^2}$

velocity: $v_n = \frac{ke^2}{n\hbar}$

For a real system m should be the center of mass m' where $m' = \frac{mM}{m+M} = \frac{M}{1+M/m}$

Then $r'_n = \frac{n^2\hbar^2}{ke^2 m'} = n^2\frac{m_e}{m'}a_0$ and $E'_n = -\frac{m'}{2n^2}(\frac{ke^2}{\hbar})^2 = -\frac{m'}{m_e}\frac{E_1}{n^2}$

Hydrogen like ions: one electron bound to Ze nucleus: $v_n = \frac{ke^2}{n\hbar}$, $E_n = -\frac{mZ^2}{2n^2}(\frac{ke^2}{\hbar})^2$,

$r_n = \frac{n^2\hbar^2}{kZe^2 m}$

6) Particles as waves

p = h/λ or λ = h/p

$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240eV\bullet nm}{\sqrt{2mc^2K}}$

$ke^2 = 1.44nm(eV)$

de Broglie wave length λ = h/p

Uncertainty principle

$\Delta k\Delta x \geq 1/2$ and $\Delta\omega\Delta t \geq 1/2$ which can also be written as
$\Delta p\Delta x \geq \hbar/2$ and $\Delta E\Delta t \geq \hbar/2$

7) Schrödinger Equation in one Dimension

General properties of Schrödinger's Equation: Quantum Mechanics

Schrödinger Equation (time dependent) $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U\Psi = i\hbar\frac{\partial\Psi}{\partial t}$

Standing wave $\Psi(x,t) = \Psi(x)e^{-i\omega t}$

Schrödinger Equation (time independent) $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U\psi = E\psi$

Normalization (one dimensional) $\int_{-\infty}^{+\infty} \Psi^*(x,t)\Psi(x,t)\,dx = 1$

For a constant potential E>U $\psi(x) = Ae^{-ikx} + Be^{+ikx}$ with $k = \sqrt{\dfrac{2m(E-U)}{\hbar^2}}$

E<U $\psi(x) = Ae^{-\eta x} + Be^{+\eta x}$ with $\eta = \sqrt{\dfrac{2m(U-E)}{\hbar^2}}$

Operators: $\hat{\mathbf{p}} = -\mathbf{i}\hbar\dfrac{\partial}{\partial x}$, $\hat{\mathbf{E}} = -i\hbar\dfrac{\partial}{\partial t}$, $\hat{\mathbf{K}} = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$, $\hat{\mathbf{H}} = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2} + U$

Eigenfunctions and Eigenvalues: $\hat{\mathbf{p}}\Psi = p\psi$ then p is the eigenvalue and Ψ is an eigenfunction.

7-1) One Dimensional Quantum Systems

a) Particle in a one-dimensional box, with U=0 for 0<x<L;

Wave functions $\psi(x) = \sqrt{\dfrac{2}{L}}\sin(n\pi x/L)$ with n= 1, 2, 3, ----

Energies $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$

b) Simple Harmonic Oscillator: $U=kx^2/2$

Allowed Energies $E_n = (n+1/2)\hbar\omega_c$ with $\omega_c = \sqrt{\dfrac{k}{m}}$

First few wave functions

$$\Psi_0(x) = A_0 e^{-\frac{x^2}{2b}}$$

$$\Psi_1(x) = A_1 \frac{x}{b} e^{-\frac{x^2}{2b}} \qquad \text{with } b = \sqrt{\frac{\hbar}{m\omega_c}}$$

$$\Psi_2(x) = A_2\left(1 - \frac{2x^2}{b^2}\right)e^{-\frac{x^2}{2b}}$$

c) Tunneling: Barrier of height U_0 with Kinetic Energy E, where within the barrier $E<U_0$

Within the classically forbidden region the wave function is $\Psi(x) = Ae^{+\alpha x} + Be^{-\alpha x}$

where $\alpha = \sqrt{\dfrac{2m(U_0 - E)}{\hbar^2}}$

The Tunneling probability $T \approx e^{-\alpha L}$

d) General properties of wave functions

The wave function and its derivative must be continuous.

In the classically forbidden region (E<U) the wave function curves away from the axis, $\Psi \to 0$ exponentially as wave penetrates the classically forbidden region.

In the classically allowed region (E<U) the wave function curves toward axis and oscillates

Constants

$c = 2.998 \times 10^{+8}$ m/s

$h = 6.626 \times 10^{-34}$ J.sec $= 4.136 \times 10^{-15}$ eV.sec

$\hbar = 1.055 x 10^{-34} J.s = 6.582 x 10^{-16} eV.s$

$k = \frac{1}{4\pi\varepsilon_0} = 8.988 x 10^9 N.m^2/C^2$

$m_e = 9.109$ x $\times 10^{-31}$ kg

$m_e c^2 = 0.511$ MeV

$m_p = 1.673 \times 10^{-27}$ kg

$m_p c^2 = 938.28$ MeV

$m_n = 1.675 \times 10^{-27}$ kg

$m_n c^2 = 939.57$ MeV

$m_p = 1836 m_e$

$m_n = 1839 m_e$

1u = 931.5 MeV/ c^2

nm = 10^{-9} m

$e = 1.6 \times 10^{-19}$ coul

$k_B = 8.617 \times 10^{-5}$ eV/K

$eV = 1.6 \times 10^{-19}$ J

$N_A = 6.022 x 10^{23}$ objects/mole

binomial expansion: $(1 + x)^n = 1 + nx + n(n-1)x^2/2! + n(n-1)(n-2)x^3/3!$

μm=10^{-6}m ,nm=10^{-9}m, pm=10^{-12}m, fm=10^{-15}m

K = 10^3, M = 10^6, G = 10^9

Useful relationships:

$hc = 1240 eV \bullet nm$

$\hbar c = 197 eV \bullet nm$

$ke^2 = 1.44 eV \bullet nm$

$\frac{ke^2}{\hbar c} = \frac{1}{137}$

$$R = \frac{m\left(ke^e\right)^2}{4\pi c\hbar^3} = \frac{mc^2\left(ke^e\right)^2}{4\pi\left(\hbar c\right)^3}$$

$R = 0.011 nm^{-1}$