Math 135 Cheat Sheet for Final Exam

Set Theory Notation		
empty set	$\varnothing$	$\{\ \}$
subset	$A \subseteq B$	$\forall x\colon x \in A \to x \in B$
proper subset	$A \subset B$	$A \subseteq B \wedge \exists y \in B\colon y \notin A$
superset	$A \supseteq B$	$B \subseteq A$
proper superset	$A \supset B$	$B \subset A$
set equality	$A = B$	$A \subseteq B \wedge B \subseteq A$
union	$A \cup B$	$\{x \mid x \in A \vee x \in B\}$
intersection	$A \cap B$	$\{x \mid x \in A \wedge x \in B\}$
difference	$A - B$	$\{x \mid x \in A \wedge x \notin B\} = A \cap \overline{B}$
symmetric difference	$A \Delta B$	$\{x \mid x \in A \leftrightarrow x \notin B\}$
complement	$\overline{A}$	$\{x \mid x \notin A\} = U - A$
Cartesian product	$A \times B$	$\{(a, b) \mid a \in A \wedge b \in B\}$
power set	$\mathcal{P}(A)$	$\{B \mid B \subseteq A\}$
cardinality	$\lvert A \rvert$	# of elements (if finite)

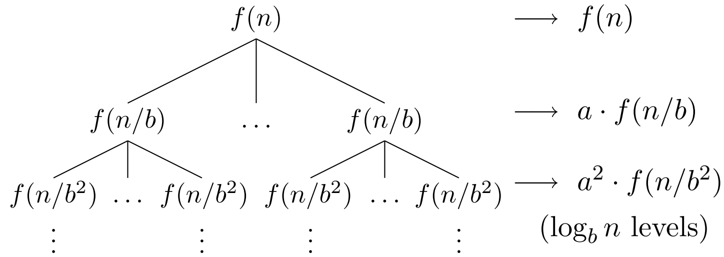
Logic	
proposition	statement which is unambiguously true or false
predicate	proposition which incorporates a variable
logical operations	and $\wedge$, or $\vee$, not $\neg$
universal quantifier	for all, written $\forall$
existential quantifier	there exists, written $\exists$
implication	if p then q, written $p \to q$
inverse of $p \to q$	$\neg p \to \neg q$
converse of $p \to q$	$q \to p$
contrapositive of $p \to q$	$\neg q \to \neg p$

Function $f\colon A \to B$

A function f from A to B associates each element $a \in A$ to exactly one element $b \in B$.

Notation	$b = f(a)$ if b is associated to a
one-to-one (or injective)	$\forall a_1, a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$
onto (or surjective)	$\forall b \in B$, $\exists a \in A$ such that $f(a) = b$
bijection	one-to-one *and* onto
inverse $f^{-1}\colon B \to A$	$\{(b, a) \mid b = f(a)\}$ (if f is a bijection)

Recursion tree for $T(n) = aT(n/b) + f(n)$



Master Theorem

Let $T(n) = aT(n/b) + O(n^k)$

If $a \geq 1$, b is an integer ≥ 1, and k a real number ≥ 0:

$$a < b^k \implies T(n) = O(n^k)$$
$$a = b^k \implies T(n) = O(n^k \log n)$$
$$a > b^k \implies T(n) = O(n^{\log_b a})$$

Asymptotic notation

$f(n) = o(g(n))$	$\forall c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) < c \cdot g(n)$
$f(n) = O(g(n))$	$\exists c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) \leq c \cdot g(n)$
$f(n) = \Theta(g(n))$	$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$
$f(n) = \Omega(g(n))$	$\exists c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) \geq c \cdot g(n)$
$f(n) = \omega(g(n))$	$\forall c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) > c \cdot g(n)$

$$f(n) = O(g(n)) \implies f(n) + h(n) = O(g(n) + h(n))$$
$$f(n) = O(g(n)) \implies f(n) \cdot h(n) = O(g(n) \cdot h(n))$$
$$f(n) + g(n) = O(\max\{f(n), g(n)\})$$
$$f(n) = O(g(n)) \text{ and } g(n) = O(h(n)) \implies f(n) = O(h(n))$$

$$\sum_{i=0}^{\infty} \alpha = \frac{1}{1-\alpha} \quad (\text{if } \alpha < 1)$$
$$\sum_{i=0}^{d} i^c = \Theta(n^{c+1}) \quad (\text{if } c \neq -1)$$
$$\sum_{i=0}^{n} c^i = \Theta(c^n) \quad (\text{if } c > 1)$$
$$\sum_{i=1}^{n} \log i = \Theta(n \log n)$$

Logarithm identities

$$\log_b(b^x) = x$$
$$b^{\log_b x} = x$$
$$\log_b x = \frac{\log_c x}{\log_c b}$$
$$\log_b(xy) = \log_b x + \log_b y$$
$$\log_b(1/x) = -\log_b x$$
$$x^{\log_b y} = y^{\log_b x}$$
$$\log_b(x^y) = y \log_b x$$

Counting	
permutation $P(n,r)$	number of ways to *list* r distinct elements from a set of size n
combination $\binom{n}{r}$	number of ways to *choose* r elements from a set of size n
Binomial theorem	$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$
coins and pirates	number of ways to distribute r identical coins to n pirates: $\binom{r+n-1}{r}$

Undirected graph $G = (V, E)$ $E \subseteq \{\{u,v\} \mid u \in V \wedge v \in V \wedge u \neq v\}$	
subgraph	$G' = (V', E')$ where $V' \subseteq V$ and $E' \subseteq E$
walk	$v_0, v_1, v_2, \ldots, v_n$ where $\{v_{i-1}, v_i\} \in E$ for all i
trail	walk with no repeated edges
path	walk with no repeated vertices
cycle	walk with no repeated vertices except $v_0 = v_n$
connected	contains a walk from any vertex to any other
acyclic	no subgraph is a cycle
tree	connected and acyclic $\implies |E| = |V| - 1$
degree sum	$\sum_{v \in V} \deg(v) = 2|E|$
Eulerian	contains a closed trail which visits every vertex
clique	set of vertices which are pairwise adjacent
independent set	set of vertices which are pairwise non-adjacent
bipartite	vertices of the graph can be partitioned into 2 independent sets
Euler's formula	in a planar graph, $|V| - |E| + |F| = 2$
number of edges in a planar graph	at most $3|V| - 6$ (by Euler's formula)